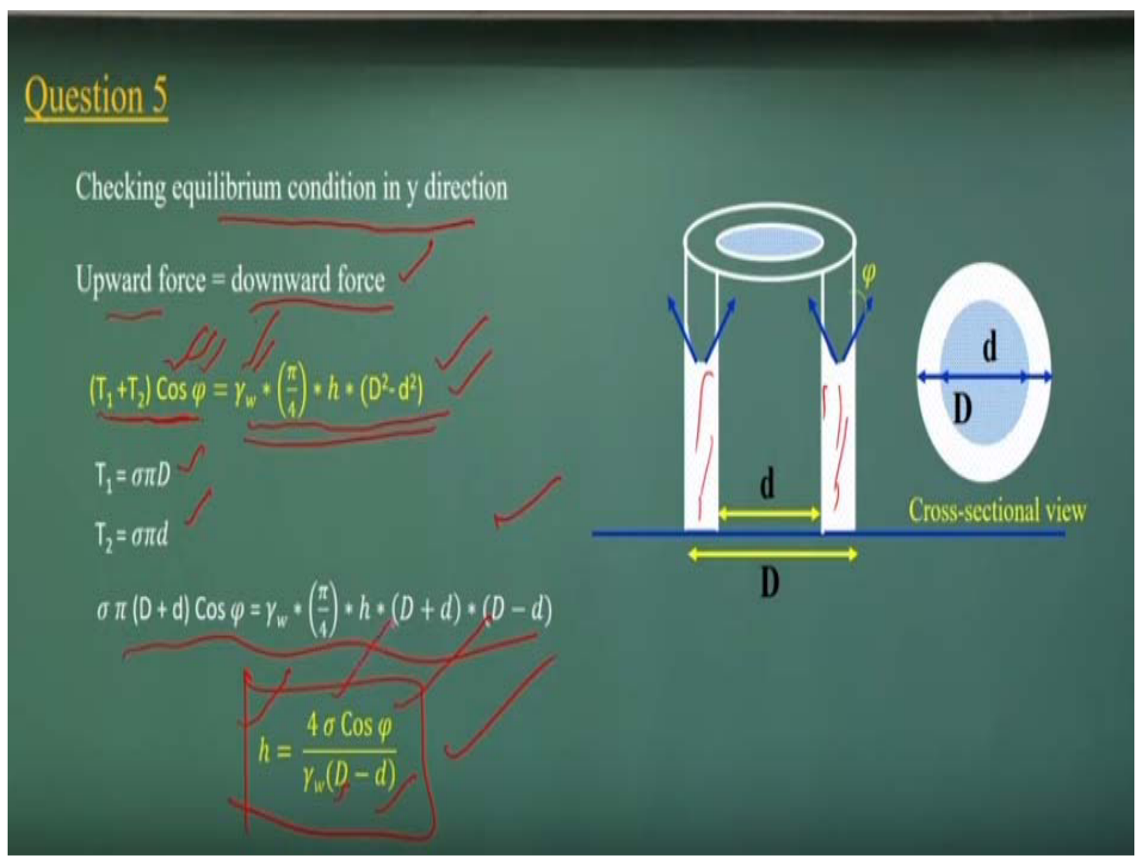

Now I have just equating this since is a equilibrium conditions in the so upward force is equal to the downward force.

Upward force = downward force

$(T_1 + T_2) \cos\varphi = \gamma w * \left(\frac{\pi}{4}\right) * h * (D^2 - d^2)$

$T_1 = \sigma\pi D$

$T_2 = \sigma\pi d$

The upward force is a surface tension force part, that what will act for a two different diameters. That what will give you this component as the upward force.

So we can compute the downward force which is the weight of the fluid. That what we confined by this the capillary rise. That what will be

$\sigma \pi (D + d) \cos\varphi = \gamma w * \left(\frac{\pi}{4}\right) * h * (D + d) * (D - d)$

$$h = \frac{4 \sigma \cos\varphi}{\gamma w (D - d)}$$

That what will give us the weight which is the weight of the fluid acting downwards, the upper part. So if I just rearrange these terms the finally, I will get this ones.

That is what very basic way I will get it the relations between the capillarity height angle of contact and these two are the diameter of annular systems where you will have a and sigma stands for surface tensions. This is a simple derivation what we have done it.

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Now let us before coming to another 5 questions to solve this is what the photographs what you can see it, I attended this MoU ceremonies held at Kyoto University in Japan. What I am to see that if I try to understand it how, what the success story behind a Japanese which is to develop after the World War II, I can visualize they have the three human characteristics that what helped them to improve their economic conditions when after the World War I. One is no doubt is hard work, truthfulness, and the punctuality.

But another things what I want to tell you that we always not have a habit to maintain a diary. But the any of Japanese if you look it, they are very good in maintaining a diary. That is the reasons they are look it, keep it their brain is free. So they are very particular to maintain a diary, the planning the activities very systematic ways.

So always if you look a Japanese, he always maintain a diary to noting it the calendars, work plan, job activities, all they noted on these and they carry the diary wherever locations they go it. So that what is another characteristics what I observed it while I interacting with Japanese groups, with whom I have been working on rainfall data analysis in northeast regions.

So what am I to say that in a fluid mechanics or any of the subjects, we always try to remember the formulaes. Please do not remember the formulaes, you try to maintain a diary for that. So slowly, you can understand those equations and try to apply this the formulaes in the right place. So as a overall the design of the course if you look it that always I encourage all of you to derive the simple equations, instead of remembering this total formula.

That should be the idea when you solve the fluid mechanics problems. Let us solve another five questions on fluid at the rest conditions.

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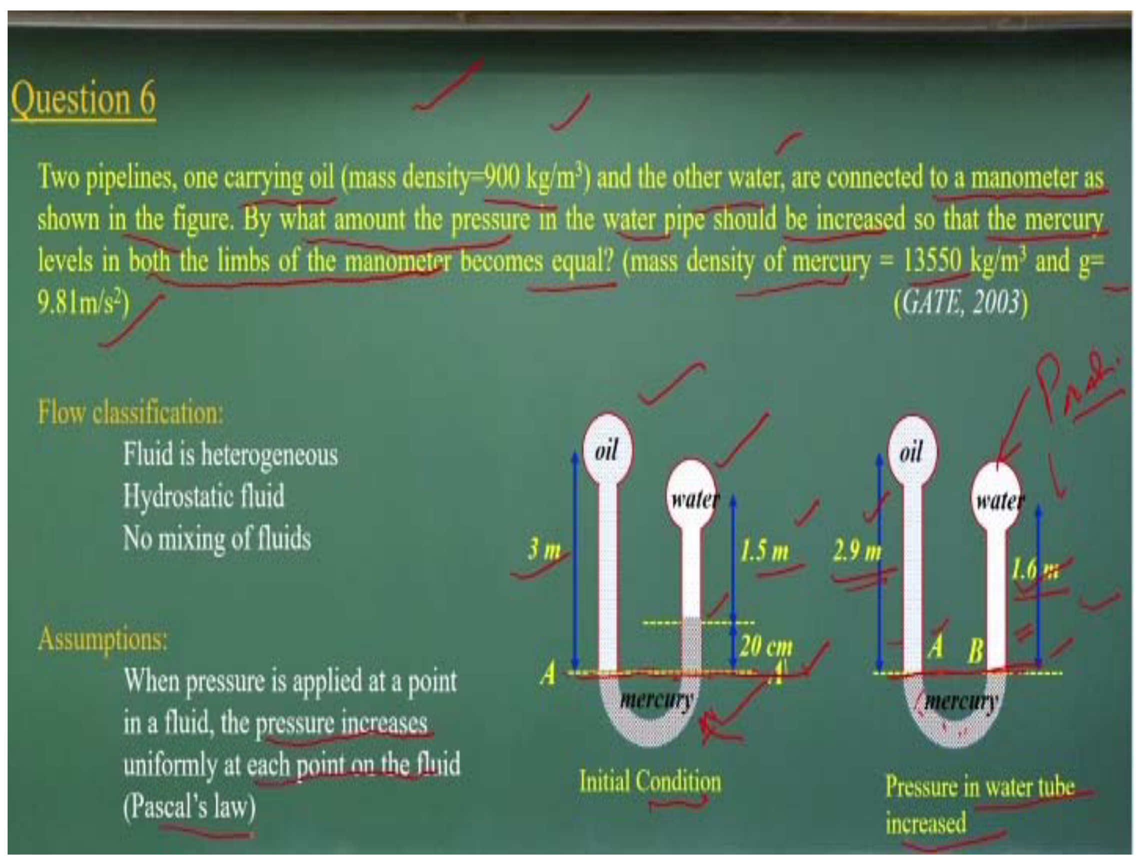

The question number 6, that is what two pipelines one carrying oil. The mass density of the oil is 900 kg per meter cube. Other one is water. It is connected to a manometer as shown in the figures. By what amount of pressure in the water pipe should be increased so that the mercury levels in the both limbs of the manometers becomes equal.

[Two pipelines, one carrying oil (mass density=900 kg/m^3) and the other water, are connected to a manometer as shown in the figure. By what amount the pressure in the water pipe should be increased so that the mercury levels in both the limbs of the manometer becomes equal? (mass density of mercury = 13550 kg/m^3 and g= 9.81m/s^2)]

The mass density of mercury is given here which is 13,550 kg/m cube, g is the acceleration due to the gravity. Now let me sketch it. So initial conditions what you have? You have oil, you have the water; it is connected to the manometers, mercury manometers which is having a 20 centimeter rise along this horizontal plane. This is a 3 meters. This is what 1.5 meter.

As we increase the pressures, the mercury manometers limbs will be the horizontal. That means, there will be a this 20 centimeter if you can understand it will be make it 10 centimeter in this side and also the 10 centimeter from this side, then it will come it to a horizontal plane. So it will be 1.6 meters, this side will be the 2.9 meters. There will be rise, there is fall.

Flow classification:

Fluid is heterogeneous

Hydrostatic fluid

No mixing of fluids

That what will be happen it. In this case will be rise it, this case will be fall it. The total mercury will come it this orientations will come when pressure in the tube is increased. So what we are going to do it for these conditions we try to find it how the pressure difference is there over this plane of A dash and the AB dash. From that, we can know it what is the additional pressure is applied here to make it the surface horizontal. So the very basic concept here just to try to understand it.

Assumptions:

When pressure is applied at a point in a fluid, the pressure increases uniformly at each point on the fluid (Pascal's law)

So pressure is applied at a point, so pressure increases uniformly at each point of the fluid. So that is what the Pascal's laws. That is what the Pascal's law.

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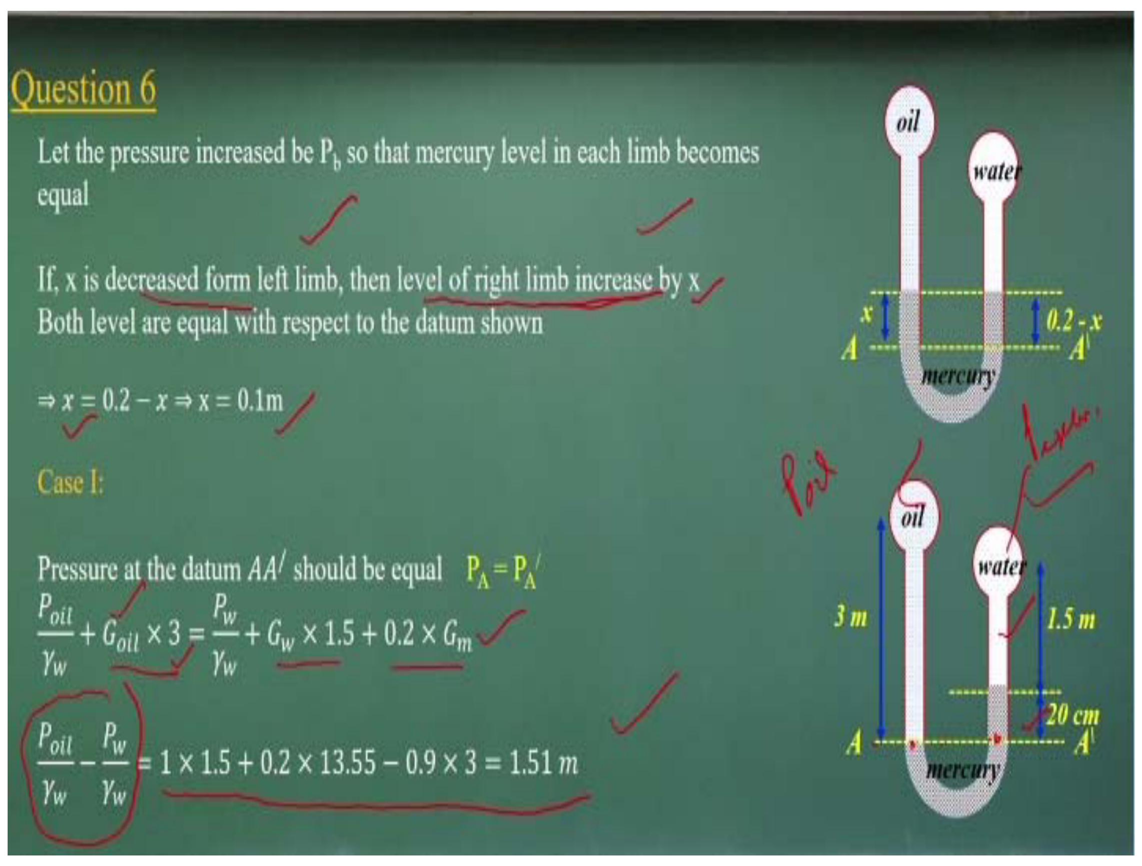

Now we are applying for the first case. As I said it to remain it the perfect levels the x will be decreased from the left limb obviously, and there will be the right limbs will be the increased by the x value. So x will be come out to be 0.1 meters.

$x = 0.2 - x \Rightarrow \mathrm{x} = 0.1\mathrm{m}$

Now I may equating the pressures P, P dash for this case. So I have the pressure at this point, pressure at this point on these horizontal surface in these manometers I am to compute what will be the pressure difference between this.

Pressure at the datum $AA^{/}$ should be equal $P_A = P_{A^{/}}$

$$\frac{P_{oil}}{\gamma_w} + G_{oil} \times 3 = \frac{P_w}{\gamma_w} + G_w \times 1.5 + 0.2 \times G_m$$

Let be consider pressure at this point is P oil. At this point is P water. So P oil, then the specific gravity into the height.

$$\frac{P_{oil}}{\gamma_w} - \frac{P_w}{\gamma_w} = 1 \times 1.5 + 0.2 \times 13.55 - 0.9 \times 3 = 1.51\ m$$

Similar way we can multiply it for water case as well as the mercury case. So if you rearrange it the pressure difference between the oil and the waters divide by the unit weight of the waters will get these values, will get this value which is equal to 1.51 meter.

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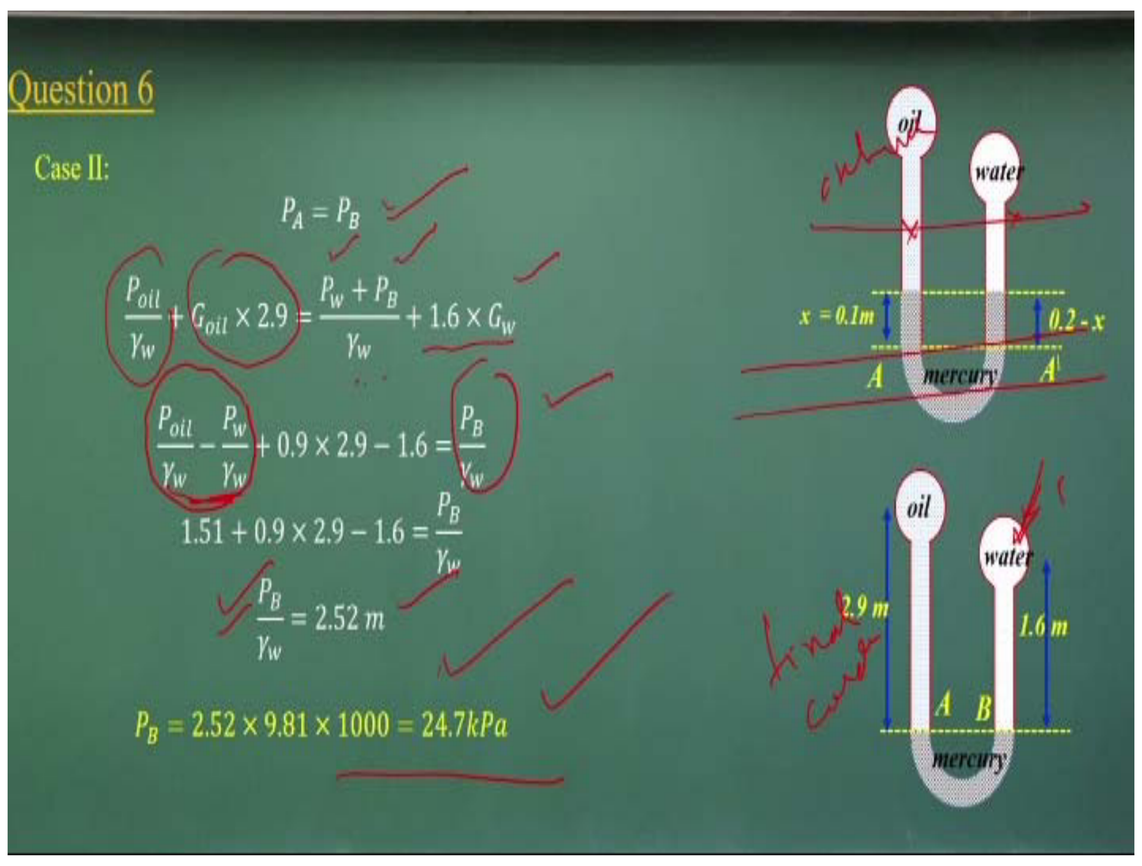

Now the second case what we will consider when it is the mercury levels at the same level okay after increasing the pressure at this point. So that what will be equal to,

$$P_A = P_B$$

$$\frac{P_{oil}}{\gamma_w} + G_{oil} \times 2.9 = \frac{P_w + P_B}{\gamma_w} + 1.6 \times G_w$$

$$\frac{P_{oil}}{\gamma_w} - \frac{P_w}{\gamma_w} + 0.9 \times 2.9 - 1.6 = \frac{P_B}{\gamma_w}$$

$$1.51 + 0.9 \times 2.9 - 1.6 = \frac{P_B}{\gamma_w}$$

$$\frac{P_B}{\gamma_w} = 2.52\ m$$

$$P_B = 2.52 \times 9.81 \times 1000 = 24.7 kPa$$

And since this component is known to us you can compute the P_B value will be 24.7 kilo Pascal. Basically try to understand it, there is a two conditions we have, the initial conditions where the manometric liquids are the different levels and then is a final conditions when we apply the additional force at this point.

So only we have applied very basic equations that in a horizontal surface whenever you take it the pressure on that horizontal surface is the same or equal pressure will act on the horizontal surface when fluid is at the rest. That the conditions what we applied it to compute what will be the additional force is acting at the water to make it the mercury level is perfectly horizontal.

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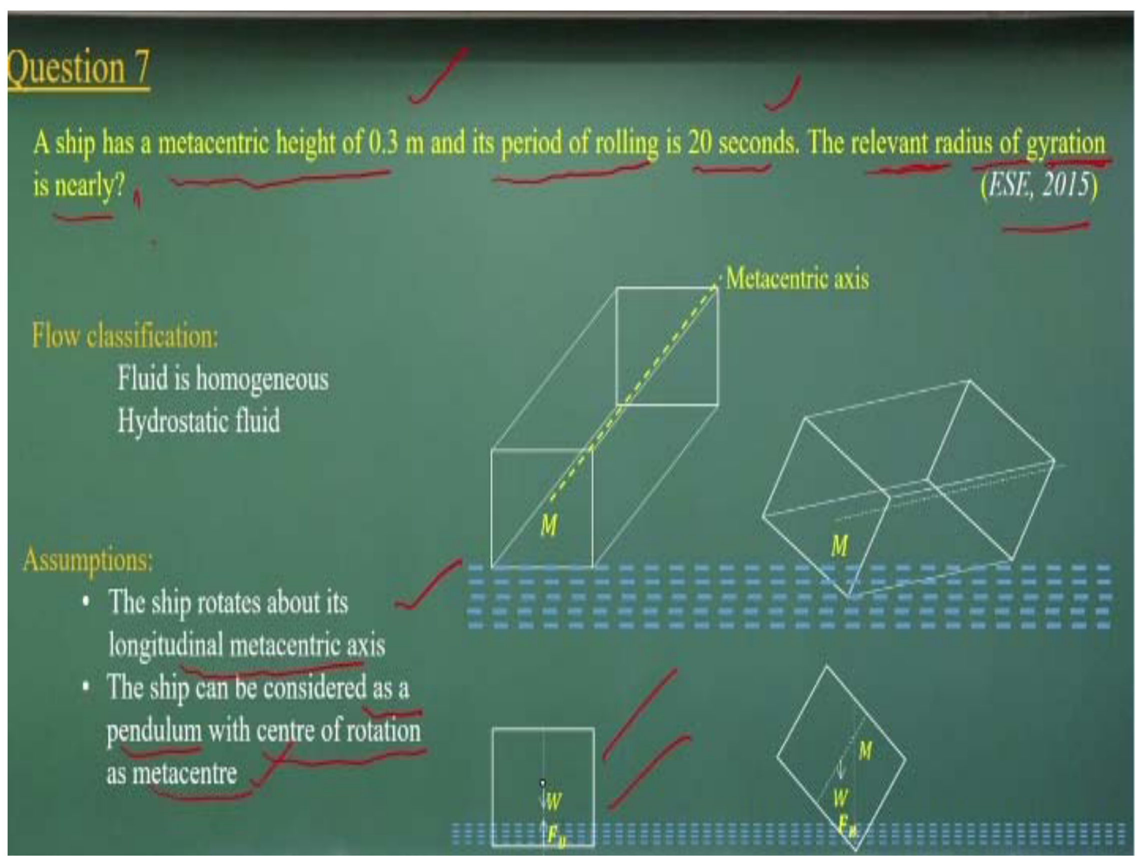

Now take it the question number 7. There is a ship of metacentric height of 0.3 meters. Its period of rolling is 20 seconds. The relevant radius of gyrations is nearly or what is the value of relevant radius of gyrations. That is what is in Engineering Service question in 2015. So as you know that there is a metacentric axis. It has known the metacentric height and it is also know it what is the time periods of rolling this ship part.

[A ship has a metacentric height of 0.3 m and its period of rolling is 20 seconds. The relevant radius of gyration is nearly?]

Flow classification:

Fluid is homogeneous

Hydrostatic fluid

Assumptions:

- The ship rotates about its longitudinal metacentric axis
- The ship can be considered as a pendulum with centre of rotation as metacentre

These are just sketched to say it the weight, the metacentric height and all the things. So the ship rotates about its longitudinal metacentric axis and it is considered as a pendulum okay as a pendulum. It has a center of rotations at the meta centers. So at the metacentric levels, this is what having the simple pendulum oscillations happening it.

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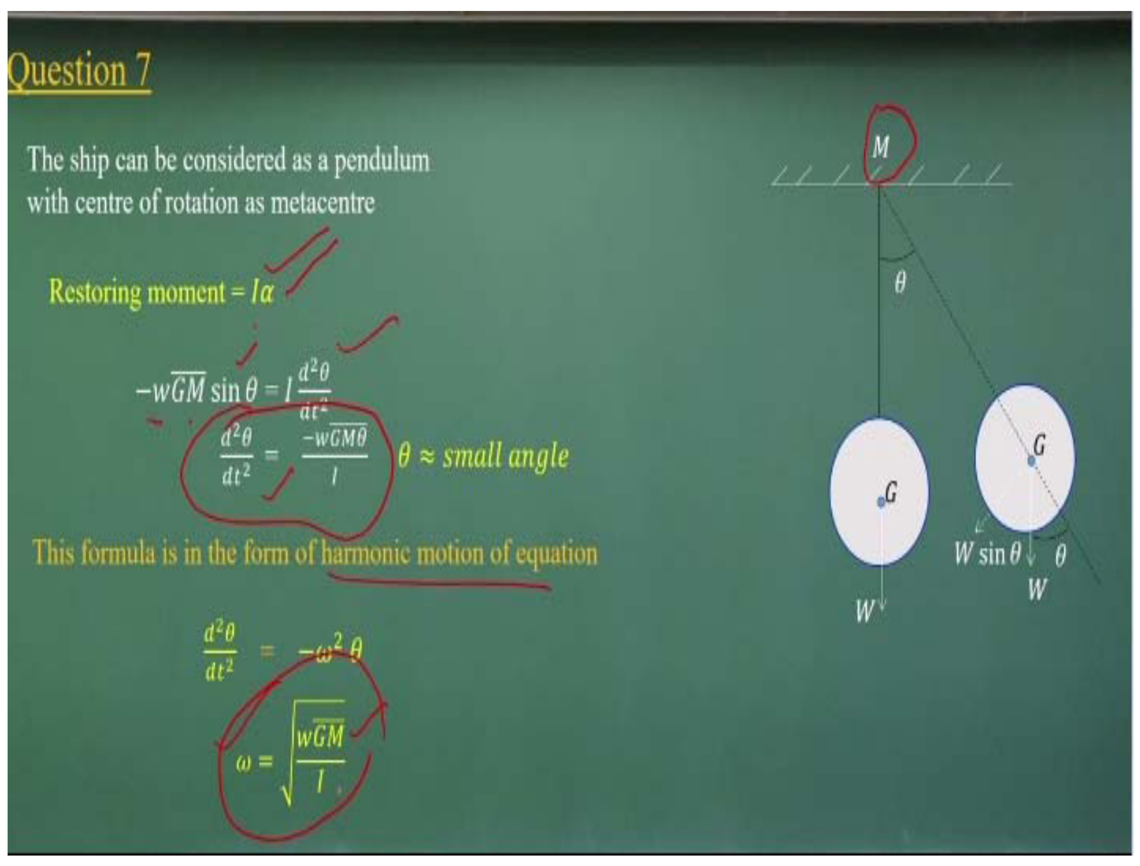

So if it is that, let me draw the free body diagrams. That means if it is the metacentric height here, this is the object is moving like this, okay? The oscillating is like a pendulum. If that we have a restoring momentum is moment of inertia into accelerations. It will be acting, what is the force torque momentum is working it that is what unit weight sin theta into GM.

Restoring moment = $I\alpha$

$-w\overline{GM}\sin\theta = I\frac{d^2\theta}{dt^2}$

$$\frac{d^2\theta}{dt^2} = \frac{-w\overline{GM}\theta}{I} \qquad \theta \approx small\ angle$$

This formula is in the form of harmonic motion of equation

$$\frac{d^2\theta}{dt^2} = -\omega^2\theta$$

$$\omega = \sqrt{\frac{w\overline{GM}}{I}}$$

And that what if you rearrange it in terms of angular oscillating component will get this part and as a harmonic components part if you look it and finally we will get it the omega in terms of unit weight GM and I is moment of inertia.

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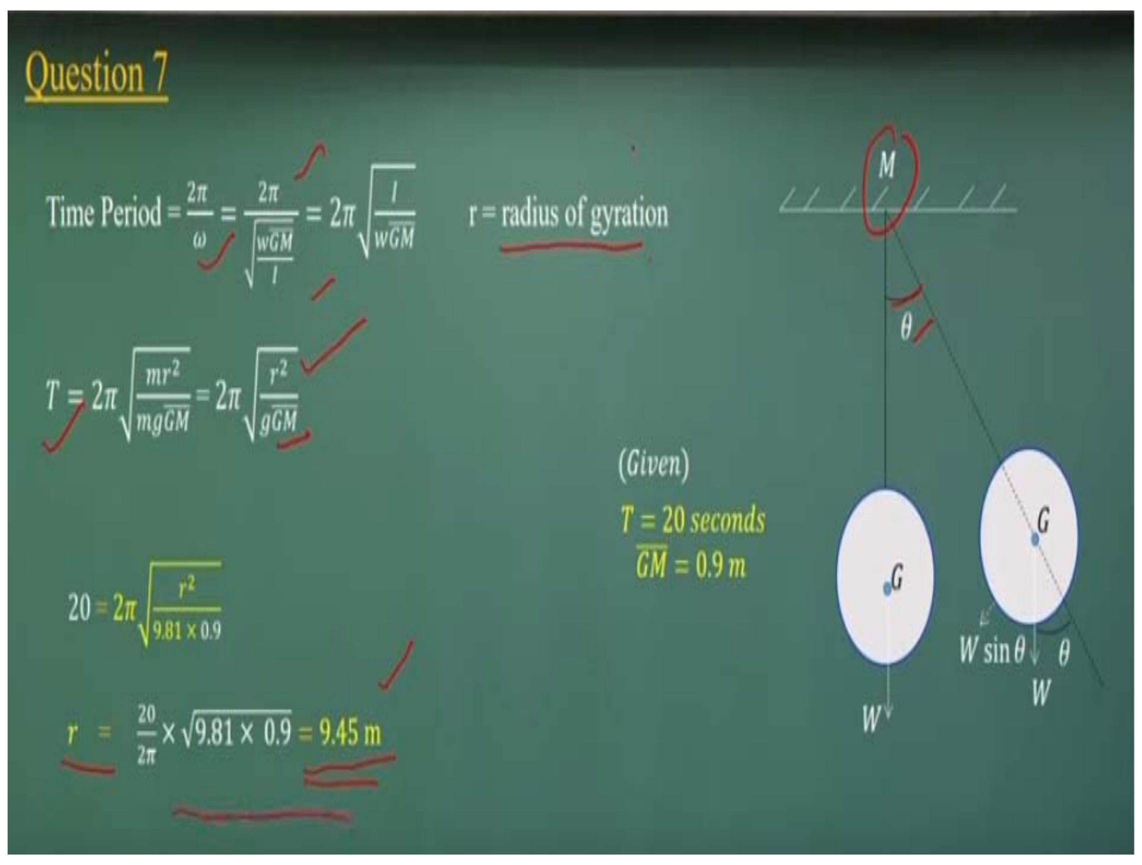

$$T = 20\ seconds$$

$$\overline{GM} = 0.9\ m$$

So the return periods or the time periods will

Time Period = $\frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{w\overline{GM}}{I}}} = 2\pi\sqrt{\frac{I}{w\overline{GM}}}$ r = radius of gyration

$T = 2\pi\sqrt{\frac{mr^2}{mg\overline{GM}}} = 2\pi\sqrt{\frac{r^2}{g\overline{GM}}}$

$20 = 2\pi\sqrt{\frac{r^2}{9.81 \times 0.9}}$

$$r = \frac{20}{2\pi} \times \sqrt{9.81 \times 0.9} = 9.45 \text{ m}$$

And it is making angle theta then what are the restoring momentum moment and what is the moment due to this oscillation. That what we are equating it to find out what will be the r value, what will be the r value.

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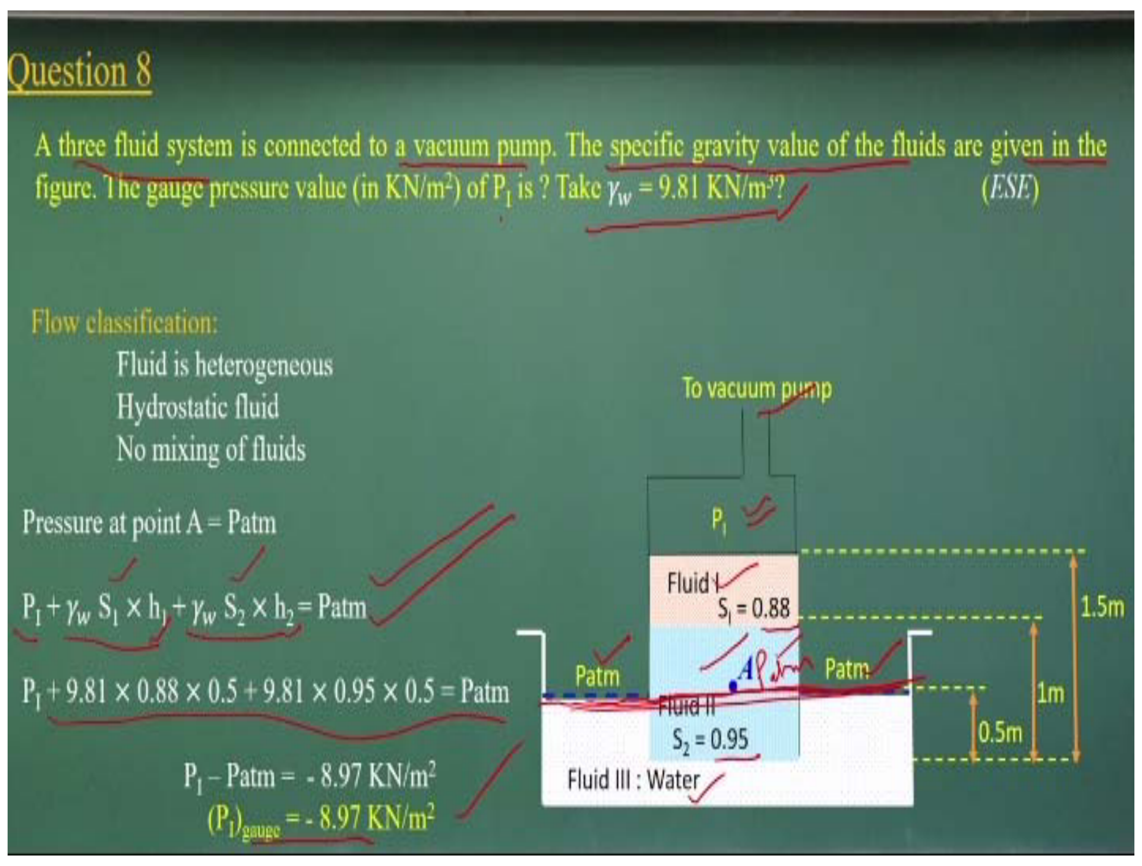

Now take it the question number 8, which is a three fluid system connected to a vacuum pump okay, where there is no pressures on this. So that is what the vacuum pump. Specific gravity values are given in these figures. Okay that is a S 1 is 0.88, 0.95 and the water which is specific gravity equal to 1. Then this gauge pressure value at P 1 what will be the pressure at the P 1 value if you consider the unit weight of the waters 9.81 kilometer per meter cube.

[A three fluid system is connected to a vacuum pump. The specific gravity value of the fluids are given in the figure. The gauge pressure value (in KN/m^2) of P_I is ? Take γ_w = 9.81 KN/m^3?]

This is very simple problems that if I take a horizontal surface here the pressure is atmospheres I can compute the pressure at this point also will be the atmosphere. So I know this present at this point is atmospheric pressure. I just equate it. If I compute the pressures from these vertical directions what is the pressure is coming that should be equal to the atmospheric pressure. That what is done it here.

Flow classification:

Fluid is heterogeneous

Hydrostatic fluid

No mixing of fluids

Pressure at point A = P_{atm}

$P_1 + \gamma_w S_1 \times h_1 + \gamma_w S_2 \times h_2 = P_{atm}$

$P_1 + 9.81 \times 0.88 \times 0.5 + 9.81 \times 0.95 \times 0.5 = P_{atm}$

$P_1 - P_{atm} = -8.97 \text{ KN/m}^2$

$$(P_1)_{gauge} = -8.97 \text{ KN/m}^2$$

P_1 is pressure at this point. As we go down the unit weight specific gravity into the height, the pressures of this fluid one then this fluid two, the unit weight, the specific gravity S_2 into h_2 should equal to the atmospheric pressure, should equal to the atmospheric pressure. Just equating that we will get the gauge pressure will be the negative of 8.79 kilometer per meter square.

So this is very easy problems. Only you have to find out where you have to equate the pressures. Where you have to take the horizontal lines. So that way you can equate the problems and solve the problems. If you do not take properly the horizontal surface to equate the pressures at two locations, then you try to do the mistakes. That what I have to tell it.

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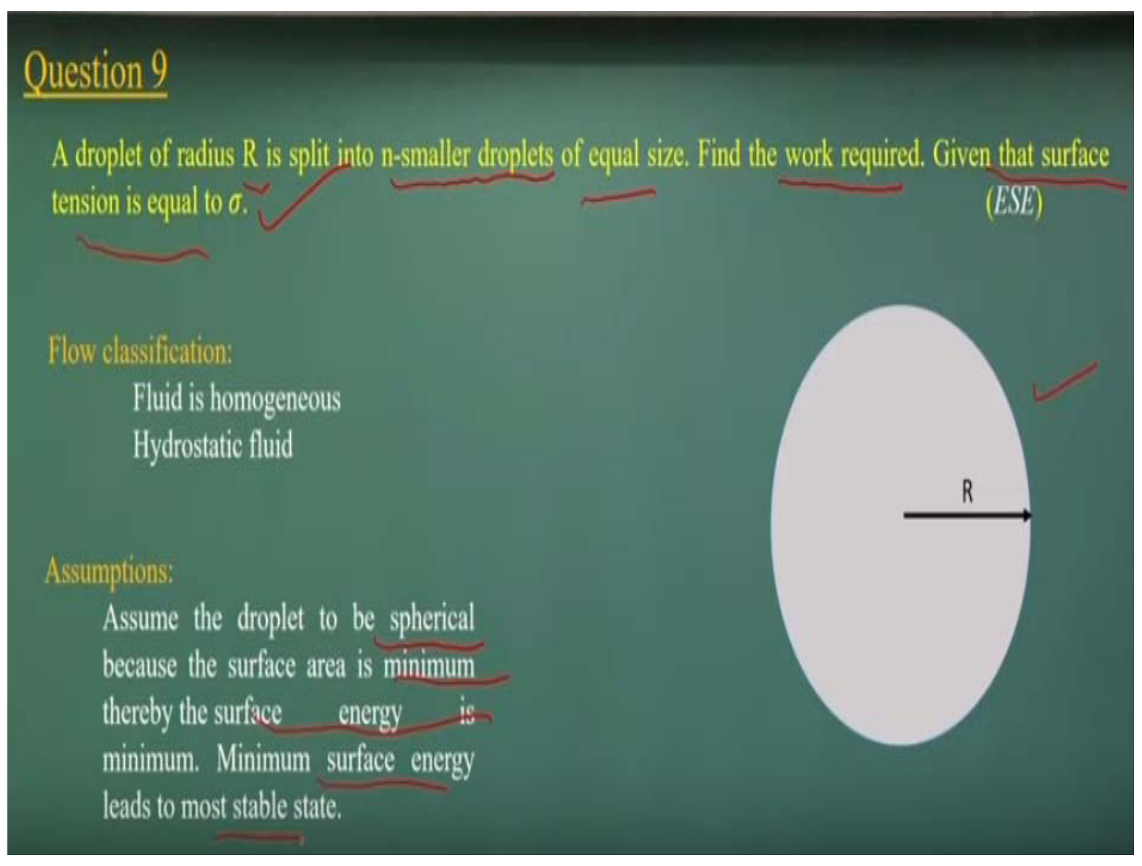

Another this is a very interesting problems is that a droplet of radius R split into n small droplets okay. So bigger droplets is split it into n number of small droplet of equal size.

Find the work required given that the surface tensions is equal to the sigma value. So that means a bigger radius R is there and which is a split it to the n small number of droplets. And that is what is happened to go for more surface areas.

[A droplet of radius R is split into n-smaller droplets of equal size. Find the work required. Given that surface tension is equal to σ.]

As its go for the more surface areas, it is need to do the work against that surface area increase of the surface area there will be more surface tension force will be act as we increase the surface area. So that is what we are trying to look it if a bigger droplet of radius R split into n number of smaller droplets then how much of extra area, surface area we are generating it and because of the extra surface area, the how much force is required to give it to split into the n number of droplets.

Flow classification:

Fluid is homogeneous

Hydrostatic fluid

That the concept here and most of the concept what we consider as equivalent to a spherical balls and surfaces in minimum that what is surface energy is minimum concept what is considered that is what is a stable consider for the any droplet.

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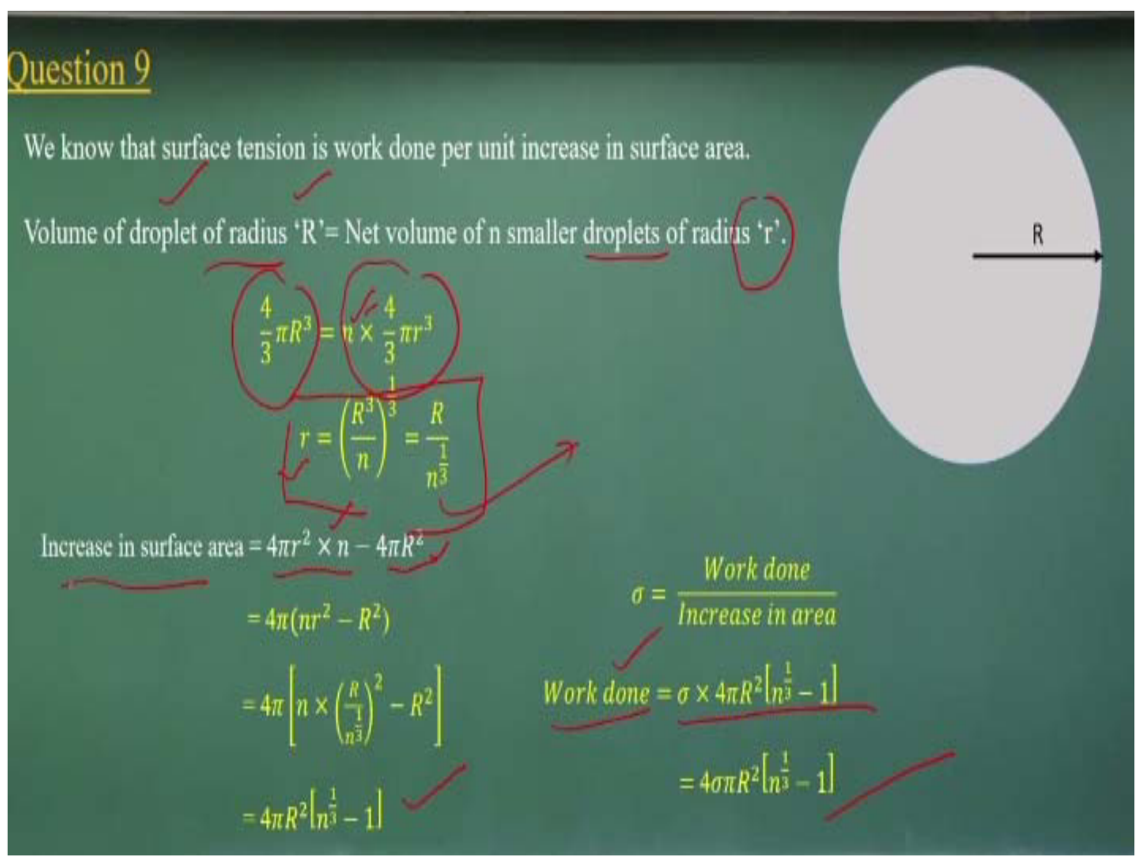

The first is that let us since the bigger droplet it is divided into n number of smaller droplet the volume should be equal okay. Of the spherical volumes what you are getting it that should be equal it. That what for the if this small r is for the smaller droplet and

bigger R represent the radius of the bigger droplet that then we will get it the volume and this side and is multiplied to find out for the n number of droplet conditions.
Volume of droplet of radius 'R'= Net volume of n smaller droplets of radius 'r'.

$$\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3$$

$$r = \left(\frac{R^3}{n}\right)^{\frac{1}{3}} = \frac{R}{n^{\frac{1}{3}}}$$

The increase in the surface area into the surface tensions that what will give us the work done for that. That is what we have computed to compute the work done part. That just we need to compute increase in the surface area. And after knowing this increasing of surface area into the surface tension force will give us the work done part. So the basically these problems does not have a much concept wise.

Increase in surface area = $4\pi r^2 \times n - 4\pi R^2$

$$= 4\pi(nr^2 - R^2)$$

$$= 4\pi\left[n \times \left(\frac{R}{n^{\frac{1}{3}}}\right)^2 - R^2\right]$$

$$= 4\pi R^2\left[n^{\frac{1}{3}} - 1\right]$$

$$\sigma = \frac{Work\ done}{Increase\ in\ area}$$

$$Work\ done = \sigma \times 4\pi R^2\left[n^{\frac{1}{3}} - 1\right]$$

$$= 4\sigma\pi R^2\left[n^{\frac{1}{3}} - 1\right]$$

Only to equate the volume of droplets and n number of small droplet volumes. Then find out what is the increase of surface area. As we know that this much of work needs to be done it to split the bigger droplet to the smaller, n number of smaller droplet because this much of additional surface force is acting. So that wave force surface tension into the area will give us the work done component.

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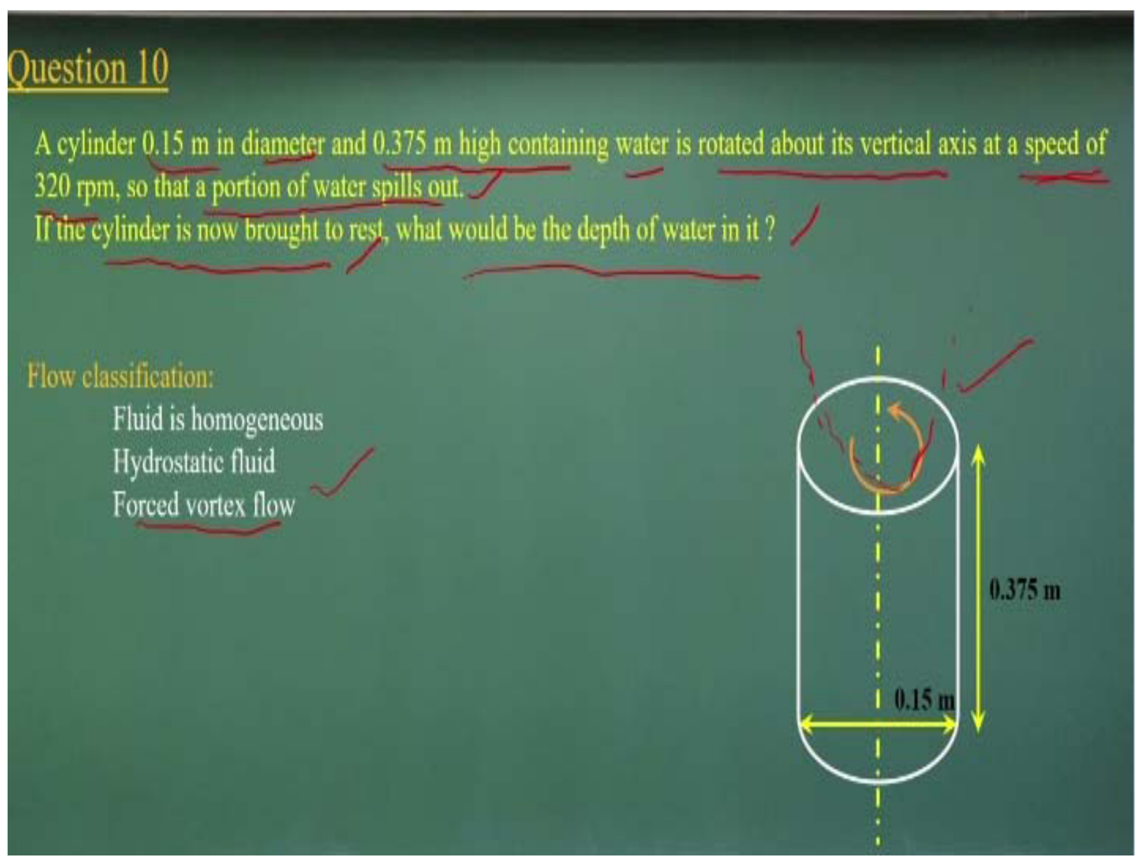

Now let me come it to very simple problems which this is the last questions what we are going to discuss is that there is a cylinders of 0.1 meter diameters, 0.375 meters high containing the water. It is rotated about the vertical axis at the speed of 320 rpm, rotations per minute so that a portion of water spills out. Okay, so if you consider a cylinder like this and you start rotating with a uniform speed of 320 rotation per minute. As you know it, it will create the free surface profile like this.

[A cylinder 0.15 m in diameter and 0.375 m high containing water is rotated about its vertical axis at a speed of 320 rpm, so that a portion of water spills out.
If the cylinder is now brought to rest, what would be the depth of water in it?]
Flow classification:

Fluid is homogeneous

Hydrostatic fluid

Forced vortex flow

So the water will be spilled. As the water spill it if it is making the cylinder is brought to the rest, what could be the depth of the water in it? So this is just a force vortex problems. Already we have derived what could be the forced vortex flow distributions part. We are just using these formulae to solve this problem.

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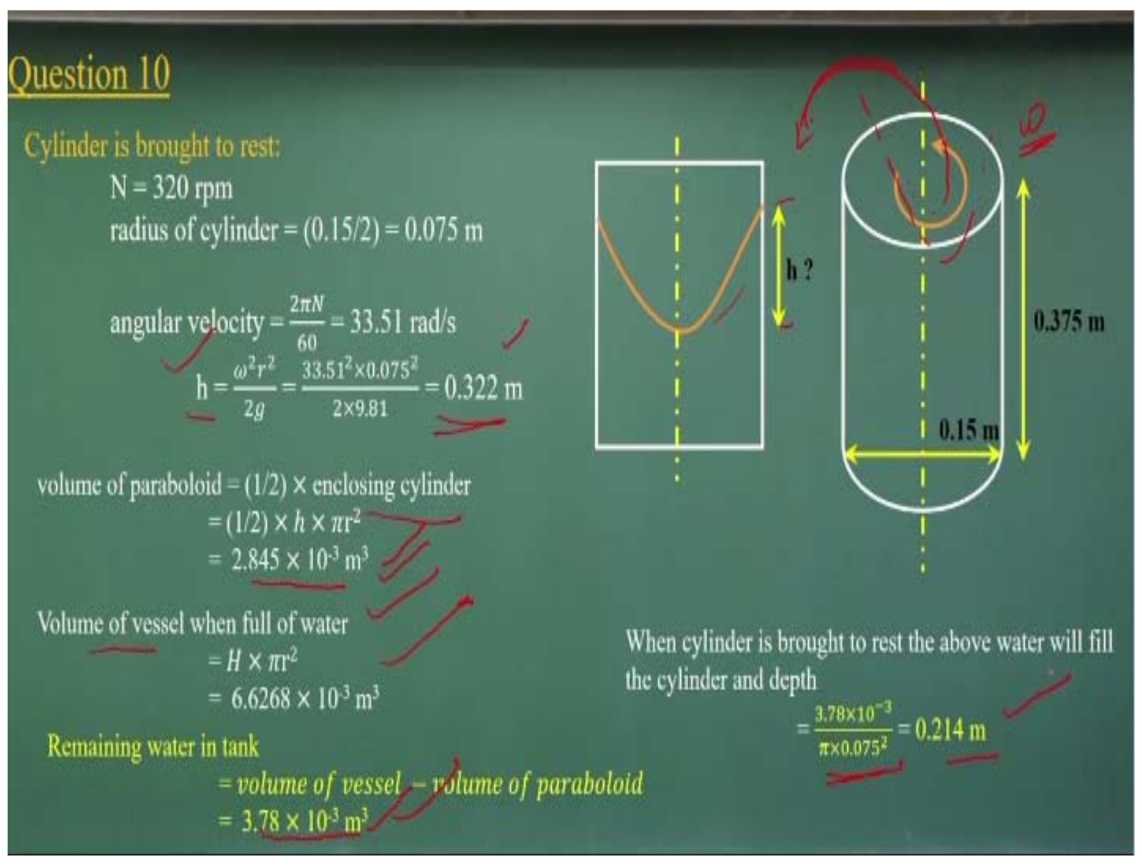

Now the first what is that if n is rotations per minute and you compute the radius,

Cylinder is brought to rest:

N = 320 rpm

radius of cylinder = (0.15/2) = 0.075 m

angular velocity = $\frac{2\pi N}{60}$ = 33.51 rad/s

That what will give us the angular velocity in terms of radian per seconds. As we are discussing it that if this is the cylindrical object, which will rotate with the omega angular velocity, you will have free surface profiles like this.

h = $\frac{\omega^2 r^2}{2g} = \frac{33.51^2 \times 0.075^2}{2 \times 9.81}$ = 0.322 m

The height h will have a relation with the angular velocity and the radius. That is what we derived in a forced vortex case. That what will it comes out to be 0.322 meters. As you know it volume of paraboloid is half of enclosing cylinders. So we know the volume that part. That means this much of volume vessels will pull water conditions and this is the volume will be spilled out.

The remaining water will be this much of volume. So divide by the surface area will get the depth what will be the remaining. Again I am to just conceptually talk about these when we are rotating a cylinder which is filled with the waters, as we have rotated with angular velocity of omega, then what will happen it, it will create a free surface like this of height h.

The remaining amount of this is the amount of the water will be spilled out from the surface. So as you know this what will be the h surface will be generated and we also know how much of volume water will be spilled out from these original amount of water we know it. Remaining amount of the volume of water we know. From there we can compute what will be the depth which is volume divided by the surface area.

volume of paraboloid = (1/2) × enclosing cylinder

$= (1/2) \times h \times \pi r2$

$= 2.845 \times 10^{-3}$ m^3

Volume of vessel when full of water

$= H \times \pi r2$

$= 6.6268 \times 10^{-3}$ m^3

Remaining water in tank

$= volume\ of\ vessel\ - volume\ of\ paraboloid$

$= 3.78 \times 10^{-3}$ m^3

When cylinder is brought to rest the above water will fill the cylinder and depth

$= \frac{3.78\times10^{-3}}{\pi\times0.075^2} = 0.214$ m

That what will be condition. So with this, let me conclude these lectures by solving these 10 problems. Thank you.